Depth extrapolation of seismic wavefields using cubic spline approximation

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SUMMARY

Depth extrapolation equation used for seismic migration is often solved by finite-difference technique. The most commonly used migration method is based upon the Crank-Nicolson implicit scheme. Some modifications to this scheme are in practice which improve the impulse response of the migration operator. In this paper we propose a different approach where the wavefield is approximated by cubic spline function. The approximation to second derivative resulting from this approach is similar to that of the 1/6th trick of Claerbout. There is another spline parameter θ which controls the nature of impulse response. The scheme is unconditionally stable for $0.5 \le \theta \le 1.0$. As the value of θ increases the dispersed evanescent energy gets more and more attenuated. The method is demonstrated by calculating the impulse response and by applying it to a synthetic data set.

INTRODUCTION

Seismic migration is a key step in imaging of the underground geological structures. For laterally varying velocity structures, the methods based upon the parabolic approximation (Claerbout 1985) of the wave equation are most common in use. Several migration methods have been proposed which use finite-difference approximation to the one way wave equation for downward extrapolation of the wavefield. Both implicit (Claerbout, 1985) and explicit (Hale, 1991) methods are in practice. Explicit methods need special care in their implementation because of the stability criteria, whereas implicit methods are unconditionally stable. In this paper we have developed an implicit scheme for the depth extrapolation of seismic wavefield using cubic spline approximation. The first part of the paper gives a mathematical description of the method. Next we demonstrate the usefulness of this method by showing the impulse responses. The method is finally tested by applying it to a synthetic data set, followed by discussion and conclusions.

MATHEMATICAL FORMULATION

Wave extrapolation equation, which is accurate for migrating dips upto 45 degree is derived from the dispersion relation

$$\frac{k_z v}{\omega} = 1 - \frac{\alpha\left(\frac{k_x v}{\omega}\right)^2}{1-\beta\left(\frac{k_x v}{\omega}\right)^2} \qquad (1)$$

where ω is the angular frequency, v is the wave velocity and k_x and k_z are horizontal and vertical wavenumbers respectively. For 45 degree accuracy α and β are 0.5 and 0.25 respectively. Accuracy for larger dips can be obtained by choosing α and β in some optimal way (Yilmaz, 1987). In terms of the retarded wavefield (Claerbout 1985) the differential equation resulting from the diffraction term is given by

$$-\frac{i\beta}{\alpha m}\frac{\partial^2}{\partial x^2}\left(\frac{\partial}{\partial z}Q\right) - \frac{im}{\alpha}\frac{\partial}{\partial z}Q + \frac{\partial^2}{\partial x^2}Q = 0 \qquad (2)$$

where $m = (\omega/v)$ and i is the square root of negative one. The other term called thin lens term is solved analytically. Equation (2) is usually solved using some finite difference approximation. Here we present a different approach for solving equation (2). Rewrite equation (2) as

$$\frac{\partial}{\partial z}\left[-\frac{i\beta}{\alpha m}\frac{\partial^2 Q}{\partial x^2} - \frac{im}{\alpha}Q\right] + \frac{\partial^2}{\partial x^2}Q = 0$$

(3)

Now approximating z derivative by forward difference approximation and ($\partial^2 Q/\partial x^2$) by second order derivatives (M_j^n) of cubic spline function $S_\Delta(x)$ (where Δ is mesh interval) interpolating $Q_j^n (j = 0,1,2,.....J)$, we get

$$-\frac{i\beta}{\alpha m}\frac{\left(M_j^{n+1} - M_j^n\right)}{\Delta z} - \frac{im}{\alpha}\frac{\left(Q_j^{n+1} - Q_j^n\right)}{\Delta z} + \theta M_j^{n+1} + (1-\theta)M_j^n = 0$$

(4)

where, $\theta \in [0,1]$ is a cubic spline parameter and Δz is increment in z-direction.

Now the continuity condition for second order derivative of cubic spline function $S_\Delta(x)$ (Ahlberg et. al., 1967) gives

$$M_{j+1}^n + 4M_j^n + M_{j-1}^n = \frac{6}{(\Delta x)^2}\left(Q_{j+1}^n - 2Q_j^n + Q_{j-1}^n\right)$$

(5)

or

$$\left(M_{j+1}^n - 2M_j^n + M_{j-1}^n\right) + 6M_j^n = \frac{6}{(\Delta x)^2}\left(Q_{j+1}^n - 2Q_j^n + Q_{j-1}^n\right)$$

$$\delta_x^2 M_j^n + 6M_j^n = \frac{6}{(\Delta x)^2}\delta_x^2 Q_j^n$$

$$M_j^n = \frac{1}{(\Delta x)^2}\frac{\delta_x^2}{\left(1+\frac{\delta_x^2}{6}\right)}Q_j^n \qquad (6)$$

where $\delta_x^2 Q_j^n = Q_{j+1}^n - 2Q_j^n + Q_{j-1}^n$.

(7)

Approximation of the second derivative by the expression given in (6) is the same as that of the 1/6 trick of Claerbout (1985). He calls it a less obvious expression that offers more accuracy at less cost, without giving an explanation for the derivation of the expression used. Here we have derived it by approximating the wavefield by cubic spline function approximation. This approximation gives better results because cubic splines provide better approximation to a given function (Ahlberg et. al., 1967). Now substituting M_j^n from equation (6) in equation (4), and multiplying by $(\Delta z)\left(1+\delta_x^2/6\right)$, we obtain

$$-\frac{i\beta}{\alpha m(\Delta x)^2}\delta_x^2 Q_j^{n+1}-\frac{im}{\alpha}\left(1+\frac{\delta_x^2}{6}\right)Q_j^{n+1}+\frac{\theta(\Delta z)}{(\Delta x)^2}\delta_x^2 Q_j^{n+1}=$$
$$-\frac{i\beta}{\alpha m(\Delta x)^2}\delta_x^2 Q_j^{n}-\frac{im}{\alpha}\left(1+\frac{\delta_x^2}{6}\right)Q_j^{n}-\frac{\theta(\Delta z)}{(\Delta x)^2}\delta_x^2 Q_j^{n} \quad (8)$$

Using (7) in (8) we obtain after some algebraic manipulations

$$AQ_{j+1}^{n+1}+BQ_j^{n+1}+AQ_{j-1}^{n+1}=\overline{A}Q_{j+1}^{n}+\overline{B}Q_j^{n}+\overline{A}Q_{j-1}^{n}$$
(9)

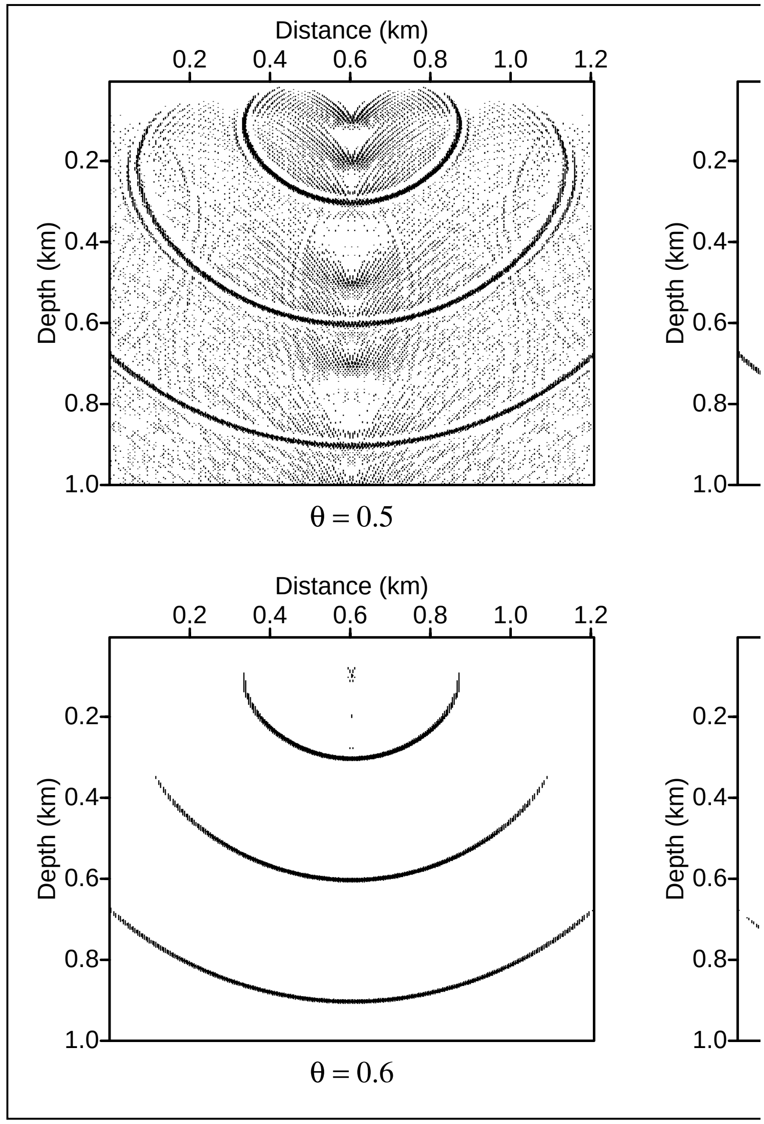

Figure 1: Impulse response of migration algorithm for different values of θ. N
as the value of θ increases. Steep dips are also attenuated with the increase i

where

$$A=\left[-\frac{i\beta}{\alpha m(\Delta x)^2}-\frac{im}{6\alpha}+\frac{\theta(\Delta z)}{(\Delta x)^2}\right]$$

$$B=\left[\frac{2i\beta}{\alpha m(\Delta x)^2}-\frac{2}{3}\frac{im}{\alpha}-\frac{2\theta(\Delta z)}{(\Delta x)^2}\right]$$

$$\overline{A}=\left[-\frac{i\beta}{\alpha m(\Delta x)^2}-\frac{im}{6\alpha}-\frac{(1-\theta)(\Delta z)}{(\Delta x)^2}\right]$$

$$\overline{B}=\left[\frac{2i\beta}{\alpha m(\Delta x)^2}-\frac{2}{3}\frac{im}{\alpha}+\frac{2(1-\theta)(\Delta z)}{(\Delta x)^2}\right]$$

Equation (9) is a tridiagonal system of equation. The solution of (9) gives the wavefield at depth (n+1) in terms of the wavefield at depth (n). This implicit scheme is unconditionally stable for $0.5 \le \theta \le 1.0$. Absorbing boundary conditions are used on the sides of the model.

IMPULSE RESPONSE

In order to test the extrapolation method described above, a 45 degree migration program was developed. The input to the migration program was a section containing three band limited Ricker wavelets in the centre of the section. The dominant frequency of the wavelets was 30 Hz. Spatial sampling of 8m and a sampling rate of 2ms was used. The migration of this data set yields the impulse responses shown in Figure 1 for different values of θ. For $\theta = 0.5$ one can observe a large amount of dispersed evanescent energy. Also at steep dips there is dispersion of low and high frequencies. For $\theta = 0.55$ the dispersed evanescent energy has been attenuated to a great extent. Also all the dips are correctly positioned. For $\theta = 0.6$ and $\theta = 0.7$ the dispersed evanescent energy is further reduced. Steep dips are also attenuated, but the remaining dips are correctly positioned along concentric semicircles. One has to decide on an optimal choice for θ, which reduces the undesired evanescent energy as well as correctly positions all the required dips. $\theta = 0.6$ seems to be the best choice from the practical point of view.

SYNTHETIC DATA EXAMPLE

Next we applied the migration method based upon the cubic spline approximation to a synthetic data set. The velocity model used for generating the synthetic data is shown in Figure 2. The model comprises of a weathered layer on top of a dipping layer and a graban like structure. Synthetic seismograms were calculated for this model for 46 source positions with a source interval of 50m and a receiver interval of 25m. A higher order finite difference modelling algorithm based upon acoustic wave equation was used with a grid spacing of 5m. A Ricker wavelet with a dominant frequency of 30Hz was used as the source function.

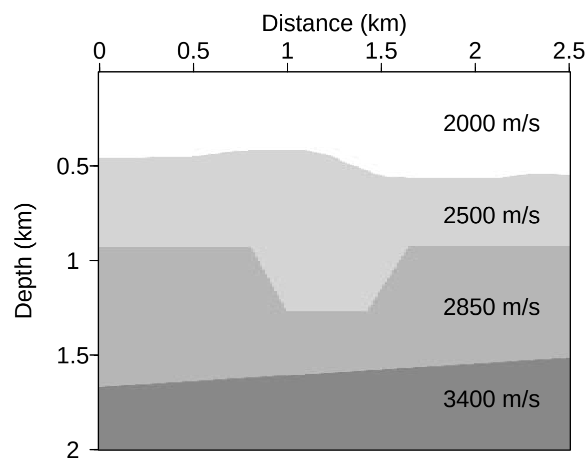

Figure 2: Velocity model used for the generation of synthetic data set.

The acoustic wave modelling code was implemented in a distributed computing environment using PVM (Parallel Virtual Machine) message passing calls (Geist et. al. 1994). For both, modelling and migration examples presented in this paper we have used 8 UltraSparc Workstations networked using a fast ethernet switch. This is a part of the facility called PARAM OpenFrame. Each UltraSparc Workstation has a 200 MHz CPU with 128 MB of RAM and Solaris operating system.

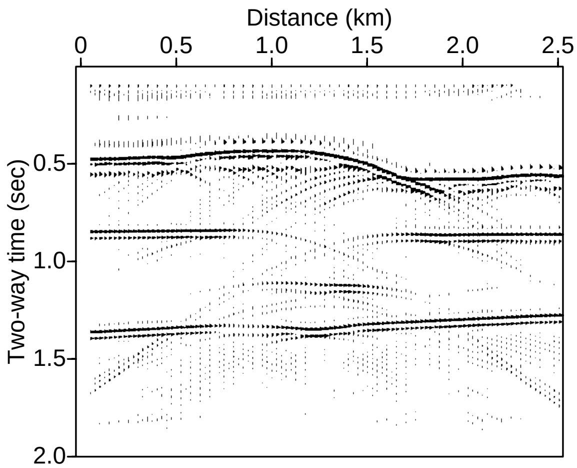

Figure 3: A CDP stacked section of the synthetic data set.

A CDP stacked section of the synthetic data set is shown in Figure 3. Next this stacked data set was migrated using the above algorithm. The migrated output sections for two different values of θ are shown in Figure 4. On both the migrated sections all the events are properly imaged. One can notice that for $\theta = 0.5$ there is more evanescent energy on the migrated section as compared to the migrated section with $\theta = 0.6$. A user can decide on the appropriate value for θ.

DISCUSSION AND CONCLUSIONS

In this paper we have developed and demonstrated an implicit scheme for the depth extrapolation of the seismic wavefield using cubic spline approximation. The approximation to the second derivative is derived from the continuity condition for the second order derivative of cubic spline function. The difference approximation to the one-way wave equation resulting from this approach has another spline parameter θ, which controls the dispersed evanescent energy. As the value of θ increases more and more evanescent energy gets attenuated. Steep dips also get attenuated with increase in the value of θ. A proper choice of this parameter helps in reducing the undesired noise on the migrated sections. This approach can be easily incorporated in 2D prestack migration algorithm as well as in 3D prestack and poststack migration algorithms.

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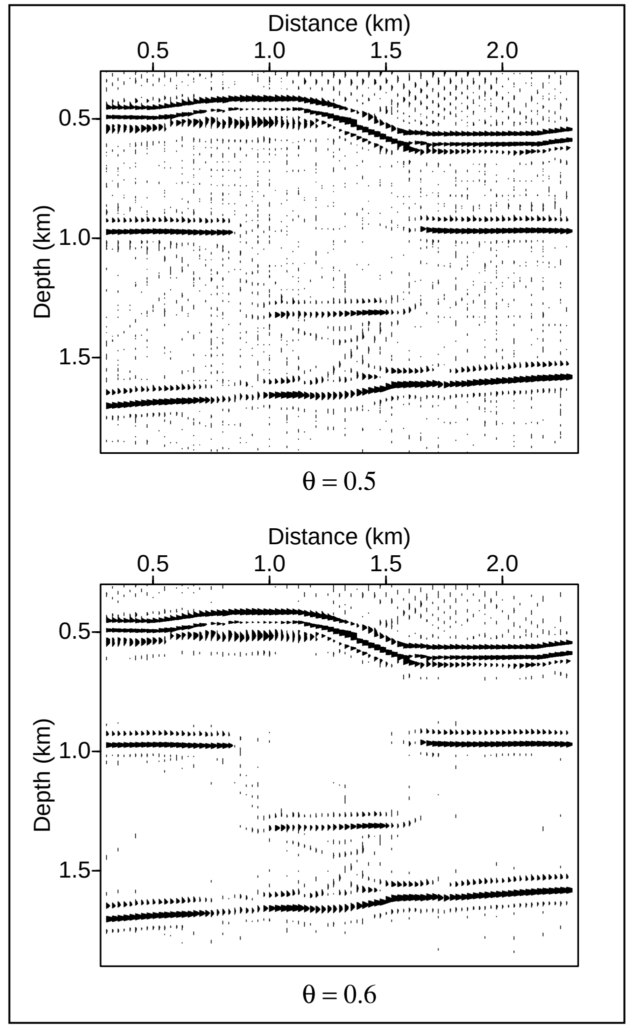

Figure 4: The migrated seismic sections of the stacked data shown in Figure 3 for $\theta = 0.5$ and $\theta = 0.6$. Notice the reduction in evanescent energy for $\theta = 0.6$.

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