Marine synthetic seismograms using elastic wave equation

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SUMMARY

Computation of synthetic seismograms for marine models is useful for the inversion and interpretation of offshore seismic data sets. In this paper we describe the implementation of an algorithm for solving 2D elastic wave equation for marine models. The resulting hyperbolic system of equations is solved on a Cartesian grid using a MacCormack finite difference scheme. The stability condition and P-wave phase velocity is independent of Poisson's ratio. Moreover, the S-wave phase velocity is insensitive to the Poisson's ratio. This allows us to use the same algorithm for liquid medium by making the Poisson's ratio equal to 0.5. The method is first tested for a simple two-layer model. It is shown that as the Poisson's ratio increases from 0.25 to 0.5 in the top layer, the converted phases are eliminated from the seismograms. Finally, synthetic seismograms and snapshots are shown for a realistic model.

INTRODUCTION

Seismic modeling is an integral part of seismic processing, as it provides us the seismic response for a given earth model (Kelly et al. 1976, Virieux 1986, Vafidis 1988, Phadke and Bhardwaj 1998). Synthetic seismograms and time slices generated by seismic modeling are also used for interpretation and inversion. Water-bottom and pegleg multiples in offshore seismic data, can significantly distort primary reflections and AVO amplitude information (Stephan, 1983). Marine synthetic seismograms for realistic models are therefore necessary in order to understand the behavior of these distortions. A homogeneous formulation that treats the liquid-solid boundary explicitly cannot accommodate complex liquid-solid boundaries. In this paper we present a heterogeneous formulation of the elastic wave equation, which is also valid in liquid medium by making Poisson's ratio equal to 0.5. This is possible because this formulation does not involve the derivatives of the physical parameters. First part of the paper gives the mathematical details followed by the numerical scheme and the explanation for the validity of the solution in a liquid medium. Then we present numerical examples for a simple model as well as for a realistic model. The algorithm is implemented on a parallel machine (PARAM 10000) using domain decomposition scheme, which allows us to calculate synthetic data for large models.

MATHEMATICAL FORMULATION

The mathematical model for elastic wave propagation in 2D heterogeneous media consists of coupled second order partial differential equations governing motions in x- and z-directions

$$\rho\frac{\partial\dot{u}}{\partial t}=\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\sigma_{xz}}{\partial z} \tag{1}$$

$$\rho\frac{\partial\dot{w}}{\partial t}=\frac{\partial\sigma_{xz}}{\partial x}+\frac{\partial\sigma_{zz}}{\partial z} \tag{2}$$

and the stress-strain relations given by

$$\sigma_{xx}=(\lambda+2\mu)\frac{\partial u}{\partial x}+\lambda\frac{\partial w}{\partial z} \tag{3}$$

$$\sigma_{xz}=\lambda(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}) \tag{4}$$

$$\sigma_{zz}=\lambda\frac{\partial u}{\partial x}+(\lambda+2\mu)\frac{\partial w}{\partial z} \tag{5}$$

where u and w are horizontal and vertical displacements, $\dot{u}$ and $\dot{w}$ are the horizontal and vertical particle velocities, σ_{xx} , σ_{zz} and σ_{xz} are the stress components, λ and μ are the Lamé parameters and ρ is the density.

Instead of solving these second order coupled partial differential equations we formulate them as a first order hyperbolic system (Virieux 1986, Vafidis 1988, Dai et al. 1996) :

$$\frac{\partial Q}{\partial t}=A\frac{\partial Q}{\partial x}+B\frac{\partial Q}{\partial z} \tag{6}$$

Where,

$$Q=\begin{bmatrix}\dot{u}\\ \dot{w}\\ \sigma_{xx}\\ \sigma_{zz}\\ \sigma_{xz}\end{bmatrix},\quad A=\begin{bmatrix}0&0&\rho^{-1}&0&0\\ 0&0&0&0&\rho^{-1}\\ \lambda+2\mu&0&0&0&0\\ \lambda&0&0&0&0\\ 0&\mu&0&0&0\end{bmatrix}$$

and

$$B=\begin{bmatrix}0&0&0&0&\rho^{-1}\\ 0&0&0&\rho^{-1}&0\\ 0&\lambda&0&0&0\\ 0&\lambda+2\mu&0&0&0\\ \mu&0&0&0&0\end{bmatrix}$$

When we move from elastic to acoustic media, the value of μ becomes zero. By substituting $\mu = 0$ in the above equation we get a first order system of hyperbolic partial differential equations which governs the acoustic wave propagation.

$$Q=\begin{bmatrix}p\\ \dot{u}\\ \dot{w}\end{bmatrix},\quad A=\begin{bmatrix}0&K&0\\ \rho^{-1}&0&0\\ 0&0&0\end{bmatrix}\ \text{and}\ B=\begin{bmatrix}0&0&K\\ 0&0&0\\ \rho^{-1}&0&0\end{bmatrix} \tag{7}$$

where *p* is the negative pressure wavefield, and $K=\lambda$ is the incompressibility.

For solving the first order hyperbolic system (6), we use the method of splitting in time (Vafidis 1988). An explicit finite difference method based on the MacCormack scheme is used for the numerical solution (Mitchell and Griffiths, 1981). This scheme is fourth order accurate in space and second order accurate in time. The model discretization is based upon regular grid, where $\Delta x = \Delta z = h$ is the grid size in the x- and z-directions, respectively; Δt is the time step; j,m,k are integers such that $x = j\Delta x, z = m\Delta z, t = k\Delta t$. Sponge boundary conditions are used for attenuating the reflected energy from the left, right and bottom edges of the model (Sochaki et al. 1987). Free-surface boundary condition is used for top edge.

NUMERICAL ANALYSIS

The problem of stability consists of finding conditions under which the difference between the theoretical and numerical solutions of the difference equation remains bounded as time progresses. A finite difference method is unstable when its error grows without bound. In the present study we are using MacCormack's finite difference method which consists of a predictor and a corrector in both x- and z- directions. By substituting the predictor value into the corrector, we get a finite difference formula for x- direction that evaluates the wavefield at $(k+1/2)^{th}$ time step from k^{th} time step

$$Q^{k+1/2}_{j,m} = -\frac{p}{12}A_{j,m}(Q^k_{j+2,m} - Q^k_{j-2,m}) - \frac{7}{12}p^2(A_{j,m})^2 (Q^k_{j-2,m} + Q^k_{j+2,m}) + \frac{2}{3}pA_{j,m}(Q^n_{j+1,m} - Q^k_{j-1,m}) + \frac{8}{9}p^2(A_{j,m})^2(Q^k_{j-1} + Q^k_{j+1,m}) + (1 - \frac{57}{36}p^2(A_{j,m})^2)Q^k_{j,m} \tag{8}$$

where p = k/h. A similar formula is obtained for the z-direction that evaluates the wavefield at $(k+1)^{th}$ time step from $(k+1/2)^{th}$ time step. The wave propagation takes place by alternating between x- and z- directions.

If a typical Fourier harmonic component $Q = Q_0 \exp(i\beta x + i\gamma z)$, where Q_0 is a constant vector, is substituted into the difference equation for Q^k, it is found that $Q^{k+1/2}$ is of the same form but with GQ_0 replacing Q_0. The matrix G is called the amplification matrix and is given by

$$G = I + ipA\frac{\sin\xi}{3}(4 - \cos\xi) - \frac{1}{2}p^2A^2(1 - \cos\xi)(2 - \sigma - \sigma\cos\xi) \tag{9}$$

where $\xi = \beta h, \eta = \gamma h$ and σ is a constant depending upon the direction of splitting.

The von Neumann necessary condition for the stability of a system requires the magnitude of the maximum eigenvalue of the amplification matrix to be less than one. Since system of equation is hyperbolic, matrix **A** (or **B**) can be diagonalized. Thus, for the MacCormack method, von Neumann condition is both necessary and sufficient for stability. If *a* is an eigenvalue of **A** the stability condition for this scheme (Gettllieb and Turkel, 1976) is given by

$$p \; |a_m| \; \leq \frac{2}{3} \tag{10}$$

where, $|a_m|$ is the largest eigenvalue of matrix **A.** The eigenvalues of matrices **A** and **B** are

$$\lambda_1 = 0, \lambda_2 = 0, \lambda_3 = 1, \lambda_4 = v_p, \; \lambda_5 = v_s.$$

Largest eigenvalue is v_p as $v_p \geq v_s$. Thus, the stability condition for the special case $\Delta x = \Delta z$, is

$$\frac{\Delta t}{\Delta x} \; v_p \; \leq \frac{2}{3} \tag{11}$$

The stability condition (11) is independent of the S-wave velocity v_s, or of the Poisson's ratio ν.

The governing equations for elastic wave propagation in the form of second order partial differential equations contain derivatives of the physical parameters. By formulating it into a first order hyperbolic system of partial differential equations, we avoid the derivatives of physical parameters. At a liquid-solid interface, there are discontinuities in physical parameters. Since the formulation does not involve the derivatives of physical parameters, the wave propagation can be carried out in heterogeneous media as long as we do not violate stability criterion (11).

For the phase velocity analysis we consider a plane wave with wavenumber *k*, which makes an angle θ with the x-axis. Following Bamberger et al.(1980), the quantity γ for the scheme presented here is given by

$$\gamma = \frac{3}{2}v_p\frac{\Delta t}{\Delta x} \tag{12}$$

that controls the numerical dispersion, and the quantity H defined by

$$H = \frac{\Delta x}{\lambda} \tag{13}$$

controls the number of grid points per wavelength of the plane wave. The resulting non-dimensional P-wave phase velocity is :

$$q_p = \frac{3}{4\pi\gamma H}\sin^{-1}\{-\frac{\gamma}{9}[\sin(4\pi H\cos\theta) + 8\sin(2\pi H\cos\theta)]\} \tag{14}$$

where q_p is independent of Poisson's ratio ν. Similarly, the non-dimensional S-wave phase velocity is

$$q_s = \frac{v_p}{v_s}\frac{3}{4\pi\gamma H}\sin^{-1}\{-\frac{v_s}{v_p}\frac{\gamma}{9}[\sin(4\pi H\cos\theta) + 8\sin(2\pi H\cos\theta)]\} \tag{15}$$

where q_s depends on Poisson's ratio through v_p/v_s as

$$\frac{v_p}{v_s} = \sqrt{\frac{1-\nu}{0.5-\nu}} \tag{16}$$

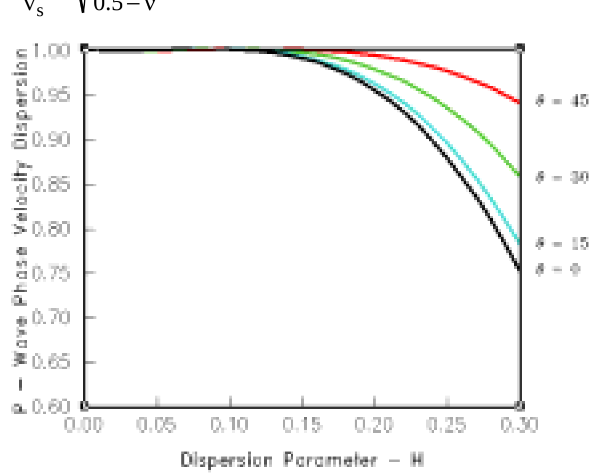

Figure 1: Dispersion curves for non-dimensional P-wave phase velocity for any Poisson's ratio with a dispersion parameter $\gamma = 0.5$. Results for different angles θ of the plane wave with respect to the x-axis are shown.

For γ = 0.5, $q_p(H)$ is plotted in Figure 1 for different angles θ. The figure is valid for any Poisson's ratio as q_p is independent of Poisson's ratio ν. Figure 2 illustrates the dispersion curves for non-dimensional S-wave phase velocity for Poisson's ratios 0.25 and 0.5. Bamberger et al. (1980) has shown that for standard finite difference schemes q_s becomes unbounded for ν=0.5. However, the scheme presented here remains stable even for a liquid medium. Therefore, propagation of elastic waves and acoustic waves across a liquid-solid interface can be carried out with the same code. No special treatment is necessary at the liquid-solid boundary and the method can be applied to complex interface geometries.

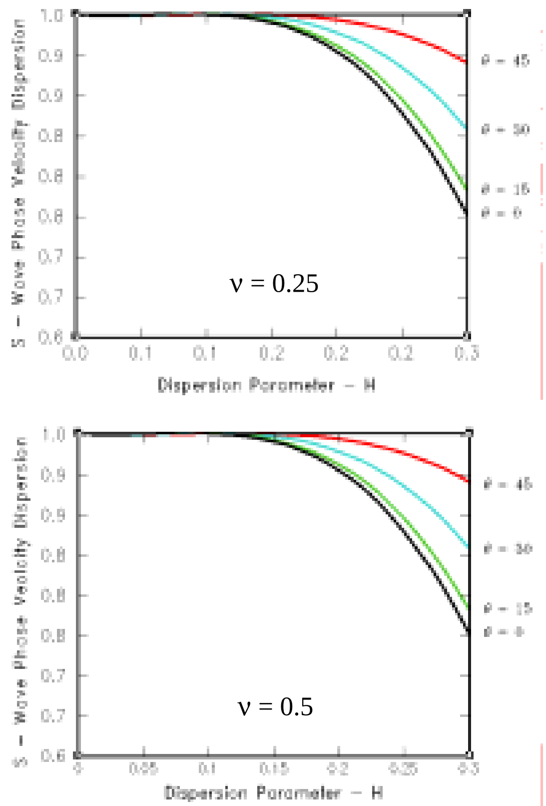

Figure 2: Dispersion curves for non-dimensional S-wave phase velocity for two different Poisson's ratios ν with a dispersion parameter γ = 0.5. Results for different angles θ of the plane wave with respect to the x-axis are shown. The upper graph is for ν=0.25 and the lower graph is for ν=0.5.

The wave propagation code is parallelized using a domain decomposition scheme and is implemented on PARAM 10000 (a cluster of SUN workstations) using MPI parallel programming environment. This implementation allows us to use this technique for generating synthetic data for large size models.

EXAMPLES

The first example presented here is a simple two-layer model (Figure 3). The synthetic seismograms for this model are calculated for different Poisson's ratios in the first medium. The source wavelet used for calculation is the second derivative of a Gaussian function with a dominant frequency of 30Hz. The synthetic seismograms for Poisson's ratios 0.25, 0.35, 0.45 and 0.5 are shown in Figure 4. Observe the PP and PS and SP reflections on these seismograms. As the Poisson's ratio increases the PS and SP reflection components reduce, and finally vanish for ν=0.5.

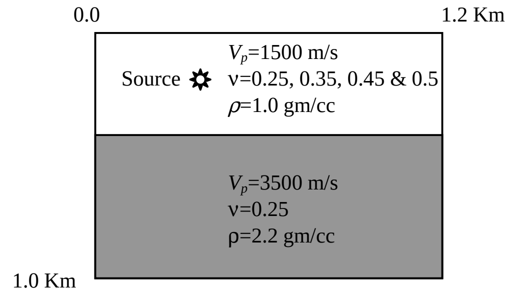

Figure 3: Velocity data and source location for the two-layer model.

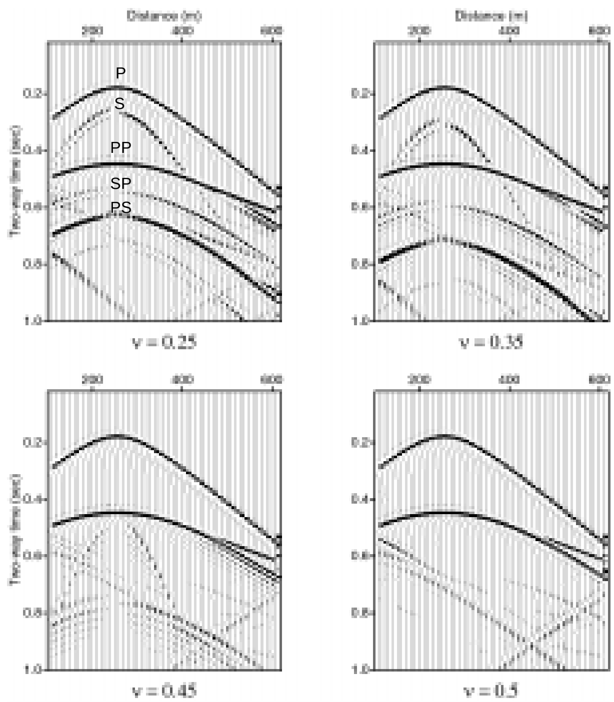

Figure 4: Evolution of vertical numerical seismograms from the solid-solid to the liquid-solid two-layer model shown in Fgure 3. Different Poisson's ratio ν are taken from 0.25 to 0.5. Absorbing boundary conditions are applied on all four sides of the model.

The P-wave velocity model used in the second example is shown in the upper part of Figure 5. There is a water layer at the top. The water bottom is quite undulating. Poisson's ratio and density in other layers are 0.5 and 2.2gm/cc respectively. The snapshots of the wave propagation through this model are shown in Figure 5. The synthetic seismogram data for this model are shown in Figure 6. A gain function is applied for display purposes. Since the free-surface boundary condition is used for the top edge, all kinds of multiples are also modeled. The example clearly demonstrates the capability of this approach for generating synthetic seismograms in realistic marine models.

CONCLUSIONS

In this paper, we have presented an algorithm for calculating synthetic seismograms in marine models using elastic wave equation. The elastic wave equation is formulated as a first order hyperbolic system. This avoids the derivatives of physical parameters. The numerical solution is obtained by using a splitting scheme and a regular Cartesian grid as opposed to a staggered grid (Virieux, 1986). The solution is valid for any Poisson's ratio, which allows us to use the same method for liquid and solid mediums. Numerical examples demonstrate the usefulness of the method. Extension of this method to 3D is quite straightforward.

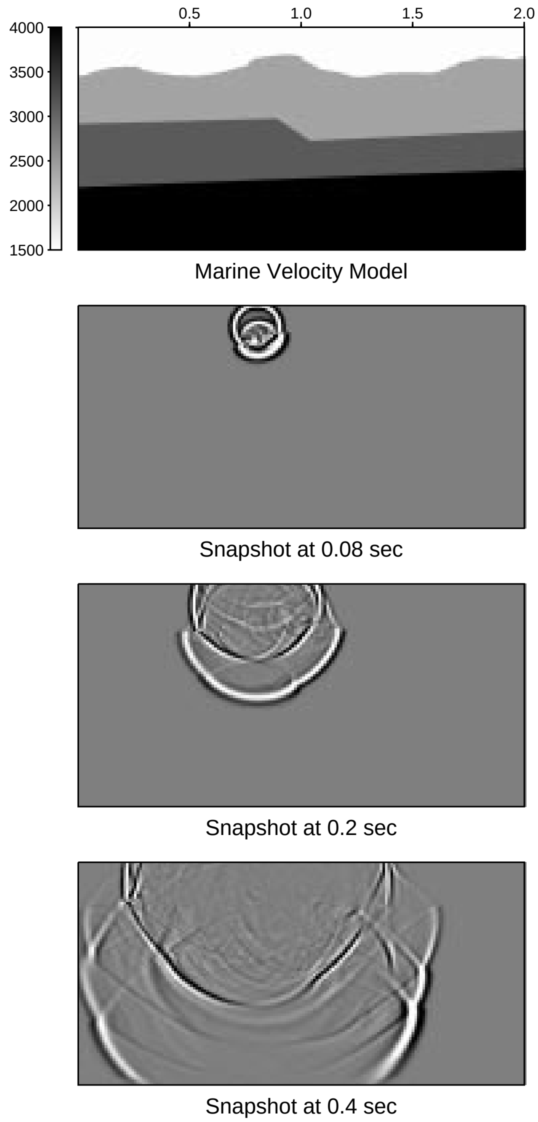

Figure 5: Snapshots of the wave propagation through the marine velocity model. Free surface boundary condition is applied on the top edge of the model and absorbing boundary conditions are applied to left, right and bottom edges of the model.

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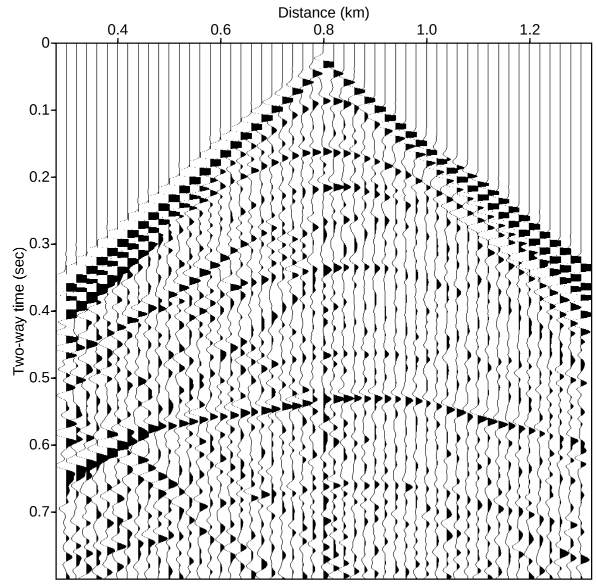

Figure 6: Synthetic seismogram for the marine model. A uniform gain function is applied for plotting purposes.

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