PVM implementation of higher order finite difference seismic modelling algorithms in a distributed computing environment

Suhas Phadke and Dheeraj Bhardwaj*
Centre for Development of Advanced Computing, Pune University Campus, GaneshKhind, Pune 411007, India

SUMMARY

Seismic wave modelling algorithms used for calculating the seismic response of a given earth model, require large computational resources in terms of speed and memory. In this paper we describe the PVM (Parallel Virtual Machine) implementation of these algorithms in a distributed computing environment. Both the acoustic and elastic wave modelling equations are formulated as a first order hyperbolic system. Numerical solution uses an explicit finite difference scheme, which is fourth order accurate in space and second order accurate in time. A domain decomposition algorithm is used for distributing the workload and the tasks communicate via PVM message passing calls. The efficiency and speed of the algorithms is tested on a cluster of SUN UltraSparc workstations.

INTRODUCTION

The finite difference methods for modelling wave propagation in the earth have gained popularity in computational seismology since their adoption in late sixties, as they offer a most direct solution to the problem expressed in terms of the basic equations and the initial and boundary conditions (Altraman and Karal 1968, Kelly et al. 1976, Virieux 1984, Virieux 1986, Phadke et al. 1991, Vafidis et al. 1992, Phadke 1994, Dai et al. 1996) . Their usage for calculating synthetic seismograms on a regular basis was not possible until the advent of high performance computers, as they require high computational speed and large memory. Recently, PVM (Parallel Virtual Machine) programming environment (Geist et al. 1994) has become popular for network computing. It has now become possible to use several computers / workstations to work in a concert to perform a single task.

This paper describes the implementation of acoustic and elastic wave modelling algorithms in a distributed computing environment. The first part of the paper gives a mathematical description, followed by the parallel implementation. Next we show the efficiency and speed of the developed modelling algorithms on a network of workstations followed by conclusions.

MATHEMATICAL FORMULATION

The propagation of both, acoustic and elastic waves in 2D heterogeneous media is formulated as a first order hyperbolic system (Virieux 1984, Virieux 1986, Vafidis 1988, Dai et al. 1996) given by

$$\frac{\partial \mathbf{Q}}{\partial t} = \mathbf{A}\ \frac{\partial \mathbf{Q}}{\partial x} + \mathbf{B}\ \frac{\partial \mathbf{Q}}{\partial z} \tag{1}$$

For acoustic wave modelling

$$\mathbf{Q} = \begin{bmatrix} p \\ u \\ w \end{bmatrix},\quad \mathbf{A} = \begin{bmatrix} 0 & K & 0 \\ \rho^{-1} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} 0 & 0 & K \\ 0 & 0 & 0 \\ \rho^{-1} & 0 & 0 \end{bmatrix} \tag{2}$$

where p is the negative pressure wavefield, u and w are the horizontal and vertical components of the velocity vector respectively, ρ is the density and K is the incompressibility.

For elastic wave modelling

$$\mathbf{Q} = \begin{bmatrix} \dot{u} \\ \dot{w} \\ \sigma_{xx} \\ \sigma_{zz} \\ \sigma_{xz} \end{bmatrix},\quad \mathbf{A} = \begin{bmatrix} 0 & 0 & \rho^{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho^{-1} \\ \lambda + 2\mu & 0 & 0 & 0 & 0 \\ \lambda & 0 & 0 & 0 & 0 \\ 0 & \mu & 0 & 0 & 0 \end{bmatrix}$$

$$\text{and}\quad \mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & 0 & \rho^{-1} \\ 0 & 0 & 0 & \rho^{-1} & 0 \\ 0 & \lambda & 0 & 0 & 0 \\ 0 & \lambda + 2\mu & 0 & 0 & 0 \\ \mu & 0 & 0 & 0 & 0 \end{bmatrix} \tag{3}$$

where $\dot{u}$ and $\dot{w}$ are the horizontal and vertical particle velocities respectively, σ_{xx} , σ_{zz} and σ_{xz} are the stress components, λ and μ are the Lamé parameters and ρ is the density.

The first order hyperbolic system (1) is solved by the method of splitting in time (Strang 1968, Vafidis 1988). An explicit finite difference method based on the MacCormack scheme is used for the numerical solution. This scheme is fourth order accurate in space and second order accurate in time. The finite difference approximation to the hyperbolic system is expressed as

$$\mathbf{Q}^{n+2} = F_x F_z F_z^{*} F_x^{*} \mathbf{Q}^{n} \tag{4}$$

where F_x, F_x^{*} are one dimensional finite difference operators approximating the one-dimensional equation

$$\frac{\partial \mathbf{Q}}{\partial t} = \mathbf{A}\ \frac{\partial \mathbf{Q}}{\partial x} \tag{5}$$

and F_z, F_z^* are one dimensional finite difference operators approximating the one-dimensional equation

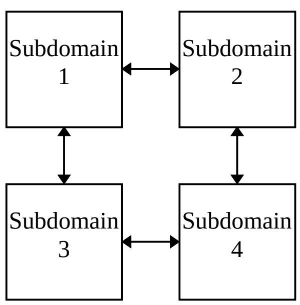
$$\frac{\partial \mathbf{Q}}{\partial t} = \mathbf{B}\,\frac{\partial \mathbf{Q}}{\partial z} \tag{6}$$

Application of each one-dimensional operator advances the wavefield by half a time step. The MacCormack scheme is a well known technique for solving first order hyperbolic systems (Mitchell and Griffiths, 1981) and consists of a predictor and a corrector. The advancement of the wavefield at any grid point to the next time level requires the knowledge of the wavefield at 9 points of the previous time level. Therefore it is a nine point difference star. Sponge boundary conditions are used for attenuating the reflected energy from the model boundaries (Sochaki et al. 1987).

PARALLEL IMPLEMENTATION

The parallel implementation of an algorithm involves the division of total workload into a number of smaller tasks, which can be assigned to different processors and executed concurrently. This allows us to solve a large problem more quickly. The most important part in parallelization is to map out a problem on a multiprocessor environment. The choice of an approach to the problem decomposition depends upon the computational scheme. Here we have implemented a domain decomposition scheme. The idea of this scheme is simple. First, the problem domain is divided into subdomains that are assigned to separate processors. Figure 1(a) shows an example of the division of problem domain into four

(a)



(b)

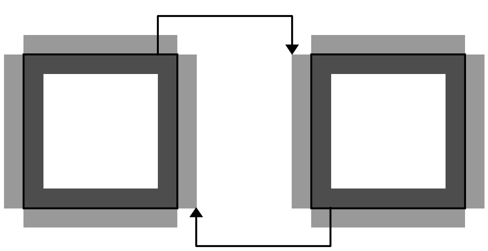


Figure 1: (a) Decomposition of model domain into subdomains (b) Communication between two adjacent tasks

subdomains. Depending upon the number of available processors and the problem, one can divide the problem domain into any number of subdomains. Since we are using a nine point difference star, the calculation of the wavefield at an advanced time level for any grid point, requires the knowledge of the wavefield at 9 grid points of the current time level. For grid points along the boundaries of the subdomain, the information about the neighbouring grid points comes from the adjacent subdomains. Therefore after each time step the subdomains have to exchange wavefield data. Figure 1(b) shows the required memory space for each 2D array of the subdomain and the communication between two adjacent subdomains. The data in the darker region is sent to the lighter region of the neighbouring subdomian using PVM message passing calls.

The two most important issues in this implementation are (1) to balance workload (2) to minimize the communication time. In a homogeneous multiprocessor environment, as in our case, the load balancing is assured if all the subdomains are of the same size. Communication is minimized by minimizing the perimeters of the subdomain boundaries.

Conceptually, PVM consists of distributed support software that executes on participating UNIX hosts on a network, allowing them to interconnect and cooperate in a parallel distributed computing environment (Geist et al. 1994). PVM offers an inexpensive platform for developing and running application. Heterogeneous machines can be used in a networked environment. The PVM model is a set of message passing routines which allows data to be exchanged between tasks by sending and receiving messages.

In the PVM implementation of the modelling codes there is a master task and there are number of worker tasks. The main job of master task is to divide the model domain into subdomains and distribute them to worker tasks. The worker tasks perform time marching and communicate after each time step. As demanded by the user the snapshot and synthetic seismogram data are collected by the master and written out on the disk.

NUMERICAL EXAMPLES

The accuracy of the wave modelling algorithms is tested by calculating the wavefield for a corner model shown in Figure 2. The physical parameters (P-wave velocity, density and Poisson ratio) inside the corner region and outside the corner

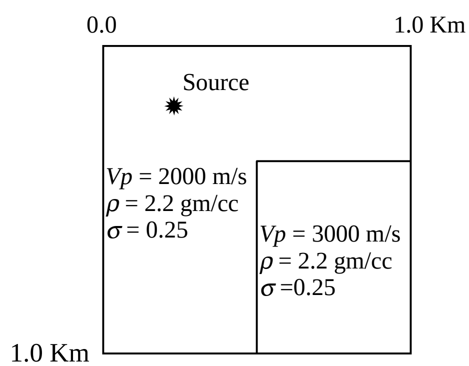


Figure 2: Velocity model for testing the wave propagation algorithms. Vp, ρ and σ denote the P-wave velocity, density and Poisson ratio in the media respectively.

region are also shown. The source pulse is a band limited Ricker wavelet with a dominant frequency of 30 Hz. The model was discretized using a grid spacing of 2m.

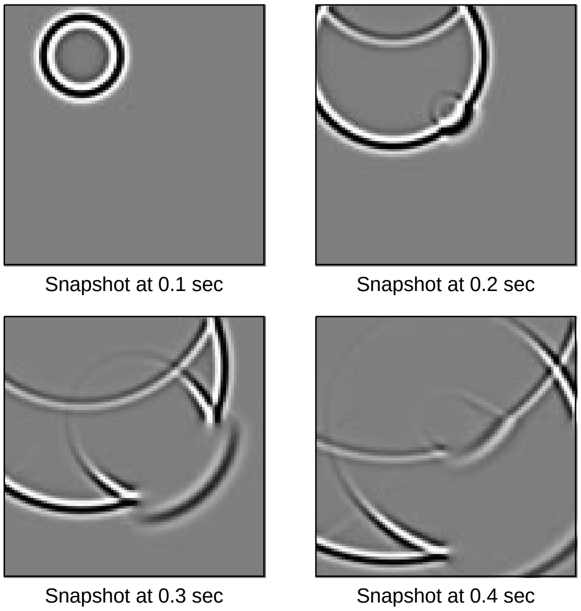


Figure 3: Snapshots of the acoustic wave propagation through a corner model. Free surface boundary condition is used at the top boundary and the absorbing boundary condition is used at all other boundaries.

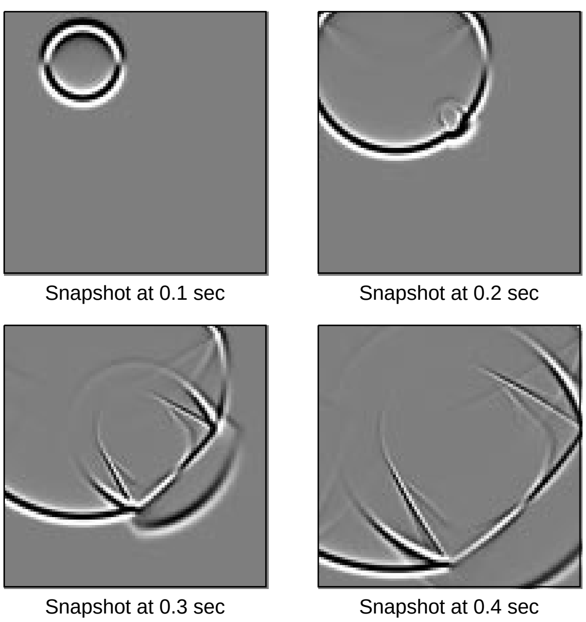


Figure 4: Snapshots of the elastic wave propagation through a corner model. The snapshots show the vertical component of the particle velocity. Absorbing boundary condition is used at all the model boundaries.

Figure 3 shows the snapshots of the acoustic wave propagation in the corner model. On these pictures one can see the direct wave, the wave reflected from the free surface, the wave diffracted from the corner, and the reflected, refracted and transmitted waves. Figure 4 shows the snapshots of the elastic wave propagation. On these snapshots the direct, reflected, refracted, converted and transmitted waves can be easily observed.

PERFORMANCE EVALUATION

Next we tested the performance of these algorithms on a cluster of Sun UltraSparc Workstations. The computing system used comprised of 6 dual CPU (200MHz) UltraSparc workstations connected via a fast ethernet switch. This a part of the facility called PARAM OpenFrame. The algorithms were tested for two different model sizes, 1200✕500 grid points and 600✕500 grid points. Figure 5 illustrates the execution times and speedup for these models. The triangles correspond to model size of 1200✕500 grid points, and the solid circles correspond to model size of 600✕500 grid points. From the graphs one can observe that for acoustic wave modelling the maximum speedup achieved on 12 CPU's is about 5 for both the model sizes. For elastic wave modelling the maximum speedup achieved on 12 CPU's is about 10. This can be attributed to the fact that there are more number of computations to be performed in elastic wave modelling. Therefore the ratio of computation to communication is higher for elastic modelling as compared to acoustic modelling.

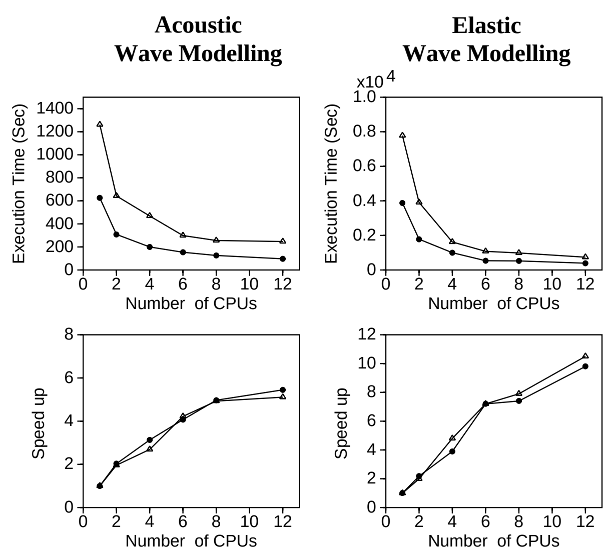


Figure 5: Execution time and Speedup of the algorithms for two different model sizes. The left graphs show the performance of acoustic wave modelling and the right graphs show the performance of elastic wave modelling. The triangles correspond to model size of 1200✕500 grid points, and the solid circles correspond to model size of 600✕500 grid points.

CONCLUSIONS

In this paper we have implemented higher order finite difference schemes for the propagation of acoustic and elastic waves in 2D heterogeneous media. The problem is formulated as a first order hyperbolic system and MacCormack finite difference scheme is used for the numerical solution. The problems are solved using a domain decomposition scheme and are implemented in a distributed computing environment (a cluster of SUN UltraSparc workstations) using PVM message passing calls. The developed codes run on a network of workstation and are fast and efficient in problem solving. This can really help in calculating synthetic seismograms for large models. Furthermore if we have to calculate survey scale data for large models, such implementation can substantially reduce the computational time. Since the codes are written using PVM message passing calls, the codes can be ported on any computing platform which supports PVM parallel programming environment. Also, since we are using a splitting technique and dealing with one dimensional operators, the extension of these algorithms to 3D is straight forward.

ACKNOWLEDGEMENTS

The authors wish to thank Department of Science and Technology (DST), Government of India, for funding this project. The authors also wish to express their gratitude to Centre for Development of Advanced Computing (C-DAC), Pune, India, for providing the computational facilities on PARAM OpenFrame and permission to publish this work.

REFERENCES

Alterman, Z. and Karal, F. C., 1968, Propagation of elastic waves in layered media by finite difference methods: Bull. Seism. Soc. America, v. 58, p. 367-398.

Dai, N., Vafidis, A., and Kanasewich E. R., 1996, Seismic migration and absorbing boundaries with a one-way wave system for heterogeneous media: Geophysical Prospecting, v. 44, p. 719-739.

Geist, A., Beguelin, A., Dongarra, J., Jiang, W., Manchek, R. and Sunderam, V., 1994, PVM: Parallel Virtual Machine, A users guide and tutorial for networked parallel computing: The MIT Press, Cambridge, Massachusetts.

Kelly, K. R., Ward, R. W., Treital, S. and Alford, R. M., 1976, Synthetic seismograms - a finite difference approach: Geophysics, v. 41, p. 2-27.

Mitchell, A. R. and Griffiths, D. F., 1981, The finite difference method in partial differential equations: John Wiley &Sons Inc.

Phadke, S., Vafidis, A., Tsingas, C. and Kanasewich, E. R., 1991, A comparative study of ray and wave methods in steam injection projects: AOSTRA Journal of Research, v. 7, p. 55-65.

Phadke, S., 1994, 2D elastic wave modelling on a transputer based parallel computer: In Supercomputing using transputers, Narosa Publishing House, New Delhi, p. 263-267.

Sochacki, J., Kubichek, R., George, J., Fletcher, W. R. and Smithson, S., 1987, Absorbing boundary conditions and surface waves: Geophysics, v. 52, p. 60-71.

Strang, G., 1968, On the construction and comparison of difference schemes: SIAM J. Num. Anal., v. 5, p. 506-517.

Vafidis, A., 1988, Supercomputer finite difference methods for seismic wave propagation, Ph.D. thesis, University of Alberta, Canada.

Vafidis, A., Abramovici, F. and Kanasewich, E. R., 1992, Elastic wave propagation using fully vectorized higher order finite differences: Geophysics, v. 57, p. 218-232.

Virieux, J., 1984, SH-wave propagation in heterogeneous media: velocity stress finite difference method: Geophysics, v. 49, p. 1933-1957.

Virieux, J., 1986, P-SV wave propagation in heterogeneous media: velocity stress finite difference method: Geophysics, v. 51, p. 889-901.