Wave equation based migration and modelling algorithms on parallel computers

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SUMMARY

This paper examines and anlyzes the dynamics of some computationally intensive seismic applications on a parallel computer. Natural divisibility into smaller tasks and localized communication pattern in seismic applications give strong motivation towards its performance on parallel machines. However we focus here on 3D poststack depth migration and 2D modelling in particular. Implementation of these algorithms in a distributed computing environment is described. Maximum degree of parallelism, efficiency and speed is achieved by reformulating the problem and proper restructuring of the code.

INTRODUCTION

Undiscovered petroleum fields are not hidden by the earths' crust so much as they are burried under a mountain of seismic data. As recoverable deposits of petroleum become harder to find and the drilling and extraction costs increase, pressure grows for more detailed imaging of underground geological structures. Current advances in 3D data acquisiton have increased the input data volume by several folds. Processing methods have also changed for high resolution processing which amounts to an increase in the computational effort. With this change in demand it has become very difficult for a processing facility built around a serial architecture machine to cope with increase in data volume and changes in processing methodology. All over the world it has been realized that parallel processing is the only answer to this challenge and we are fortunate that SDP is an ideal application for parallel architecture machines.

By nature most seismic problems carry an inherent parallelism in subdivision by sources, receivers, frequency or wave number. Thus, one must redefine the mathematical formulations and modify the serial algorithms in favour of new ones in order to obtain the full benefit of parallel computers. In this paper we describe the implementation of 3D poststack depth migration in frequency space domain and 2D acoustic and elastic wave modelling algorithms on a network of workstation using PVM (Parallel Virtual Machine) parallel programming environment.

WAVE EQUATION BASED METHODS

The wave equation based methods are widely recognised in the industry as more accurate while providing finer detailed geological features than other conventional methods. However, the wave equation based methods are computationally more expensive but suitable for currently available parallel computers.

Advanced processing techniques are distinguished primarily by their use of the wave equation, which introduces an elemnet of mathematical rigor and computational complexities not found in conventional techniques. The most common adnaced techniques include seismic migration and forward modelling.

The finite difference methods have gained popularity in computational seismology since their adoption in late sixties, as they offer a most direct solution to the problem expressed in terms of the basic equations, and the initial and boundary conditions. The finite difference solutions of both migration and modelling equations, generally require large computer memory and high computational speed. However, with the advent of supercomputers they have received special attention due to their capability of accurately imaging the complex geological structures and producing complete synthetic seismograms for reallistic earth models.

3D POST-STACK DEPTH MIGRATION

Mathematical basis for migration in ω-x domain

Seismic migration is the process through which echo sounding data recorded at the surface are mapped into images of the earths' subsurface properties (Claerbout, 1985). For laterally varying structures, the methods based upon the parabolic approximation of the wave equation are most common in use..

The migration method comprises of two steps, extrapolation and imaging. The extrapolation equation is a parabolic partial differential equation derived from the dispersion relation (Ristov, 1980)

$$k_z = \frac{\omega}{v}\left(\sqrt{1-\left(\frac{vk_x}{\omega}\right)^2} + \sqrt{1-\left(\frac{vk_y}{\omega}\right)^2} - 1\right)$$

where x, y and z are inline, crossline and depth axes respectively, k_x, k_y and k_z are the wavenumbers in x, y and z directions respectively, v is the velocity and ω is the frequency. By approximating the square root terms by continuous fraction expansion (Ma, 1981) we obtain a 45 degree approximation. By inverse Fourier Transforming in x and z we obtain the parabolic partial differential equation. This equation is numerically solved by the method of splitting, which is the basis for the onepass approach. A

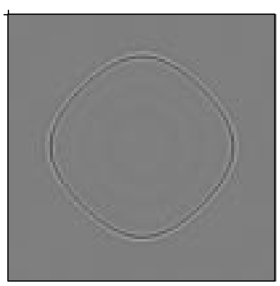
Crank-Nikolson finite difference scheme with absorbing boundary conditions on the sides of the model is used for the solution. Imaging is the summation of all the frequencies at t=0 for each depth.

Parallel Implementation

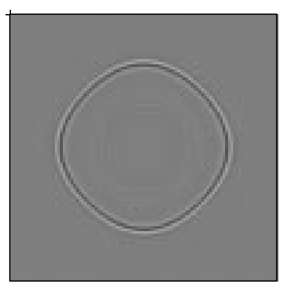
The depth migration algorithm in ω-x domain is inherently parallel in terms of frequencies. The parabolic approximation of the wave equation in frequency-space domain has decomposed the wave field into monochromatic plane waves that are propagating downwards. Therefore, each frequency harmonic can be extrapolated in depth independently on each processor and there is no need of intertask communication. One can introduce parallel task allocation into each frequency harmonic component with the ultimate goal being to have as many processors as frquencies. At each depth step all frequency components after extrapolation are summed up (Imaging Condition) to give the migrated solution. That can be done by automatic merging and reduces computational time.

Impulse response of 3D migration operator

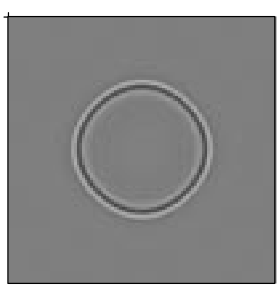
The best way to test a 3D migration algorithm is to calculate its impulse response. An



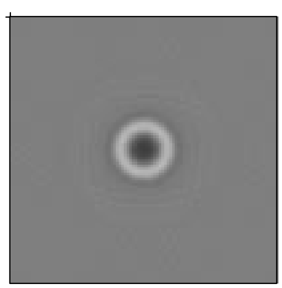
Depth Slice at 80m



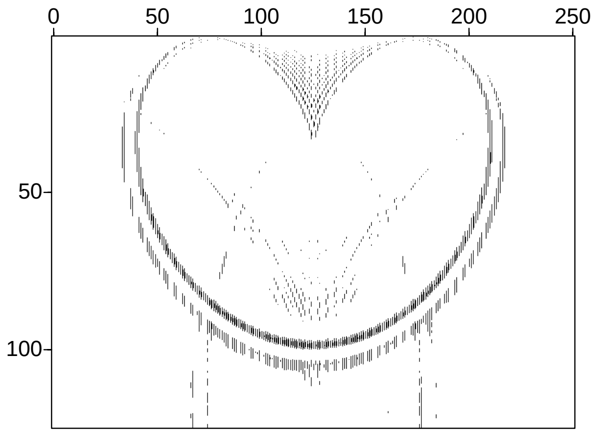
Depth Slice at 120m



Depth Slice at 160m



Depth Slice at 200m



Impulse Response

PARALLEL MODELLING ALGORITHMS

While migration is an inverse procedure, a forward procedure known as forward modelling is used to produce synthetic seismic sections. In forward modelling one starts with an assumed earth model and generate a wave field by solving wave equation. In order to gain the acceptable geological model, comparisons are often made between the synthetic and observed seismograms; errors are attributed to either inaccuracies in the model or other factors not acounted for. The model is then modified in an effort to account for the errors until adequate agreement has been reached.

A basic problem in theoretical seismology is to determine the wave response of a given model to the excitation of an impulsive source by solving the wave equations under certain simplification. When the ray tracing method is used only wave arrival times are determined. In scalar approximation, the acoustic wave equation may be solved to evaluate the waveform but only compressional waves(P-waves) are considered. A more complete appraoch is to study the vector displacement field using the full ealstic wave equation for modelling both P-waves and shear waves(S-waves). However, important wave properties such as attenuation and dispersion require a more sophisticated set of equations.

The propagation of both, acoustic and elastic waves in 2D heterogeneous media is formulated as a first order hyperbolic system (Virieux 1984, Virieux 1986, Vafidis 1988, Dai et al. 1996) given by

$$\frac{\partial \mathbf{Q}}{\partial t} = \mathbf{A}\,\frac{\partial \mathbf{Q}}{\partial x} + \mathbf{B}\,\frac{\partial \mathbf{Q}}{\partial z}$$

(1)

For acoustic wave modelling

$$\mathbf{Q} = \begin{bmatrix} p \\ u \\ w \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} 0 & K & 0 \\ \rho^{-1} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} 0 & 0 & K \\ 0 & 0 & 0 \\ \rho^{-1} & 0 & 0 \end{bmatrix}$$

(2)

where p is the negative pressure wavefield, u and w are the horizontal and vertical components of the velocity vector respectively, ρ is the density and K is the incompressibility.

For elastic wave modelling

$$\mathbf{Q} = \begin{bmatrix} \dot{u} \\ \dot{w} \\ \sigma_{xx} \\ \sigma_{zz} \\ \sigma_{xz} \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} 0 & 0 & \rho^{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho^{-1} \\ \lambda+2\mu & 0 & 0 & 0 & 0 \\ \lambda & 0 & 0 & 0 & 0 \\ 0 & \mu & 0 & 0 & 0 \end{bmatrix}$$

$$\text{and} \quad \mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & 0 & \rho^{-1} \\ 0 & 0 & 0 & \rho^{-1} & 0 \\ 0 & \lambda & 0 & 0 & 0 \\ 0 & \lambda+2\mu & 0 & 0 & 0 \\ \mu & 0 & 0 & 0 & 0 \end{bmatrix}$$

(3)

where $\dot{u}$ and $\dot{w}$ are the horizontal and vertical particle velocities respectively, σ_{xx} , σ_{zz} and σ_{xz} are the stress components, λ and μ are the Lamé parameters and ρ is the density.

The first order hyperbolic system (1) is solved by the method of splitting in time (Strang 1968, Vafidis 1988). An explicit finite difference method based on the MacCormack scheme is used for the numerical solution. This scheme is fourth order accurate in space and second order accurate in time. The finite difference approximation to the hyperbolic system is expressed as

$$\mathbf{Q}^{n+2} = F_x F_z F_z^* F_x^* \mathbf{Q}^n$$

(4)

where F_x, F_x^* are one dimensional finite difference operators approximating the one-dimensional equation

$$\frac{\partial \mathbf{Q}}{\partial t} = \mathbf{A} \ \frac{\partial \mathbf{Q}}{\partial x}$$

(5)

and F_z, F_z^* are one dimensional finite difference operators approximating the one-dimensional equation

$$\frac{\partial \mathbf{Q}}{\partial t} = \mathbf{B} \ \frac{\partial \mathbf{Q}}{\partial z}$$

(6)

Application of each one-dimensional operator advances the wavefield by half a time step. The MacCormack scheme is a well known technique for solving first order hyperbolic systems (Mitchell and Griffiths, 1981) and consists of a predictor and a corrector. The advancement of the wavefield at any grid point to the next time level requires the knowledge of the wavefield at 9 points of the previous time level. Therefore it is a nine point difference star. Sponge boundary conditions are used for attenuating the reflected energy from the model boundaries (Sochaki et al. 1987).

PARALLEL IMPLEMENTATION

The parallel implementation of an algorithm involves the division of total workload into a number of smaller tasks, which can be assigned to different processors and executed concurrently. This allows us to solve a large problem more quickly. The most important part in parallelization is to map out a problem on a multiprocessor environment. The choice of an approach to the problem decomposition depends upon the computational scheme. Here we have implemented a domain decomposition scheme.

The idea of this scheme is simple. First, the problem domain is divided into subdomains that are assigned to separate processors. Upper picture shows an example of the division of problem domain into four subdomains. Depending upon the number of available processors and the problem, one can divide the problem domain into any number of subdomains. Since we are using a nine point difference star, the calculation of the wavefield at an advanced time level for any grid point, requires the knowledge of the wavefield at 9 grid points of the current time level. For grid points along the boundaries of the subdomain, the information about the neighbouring grid points comes from the adjacent subdomains.

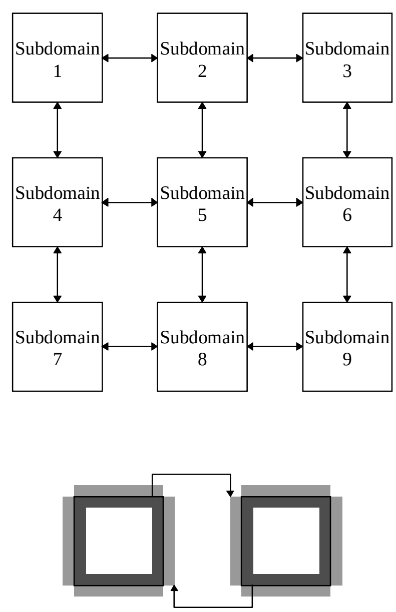


Figure 1: The upper picture shows the division of problem domian into a number of subdomians. The lower picture shows the communication between two adjacent tasks.

Therefore after each time step the subdomains have to exchange wavefield data. Lower picture shows the required memory space for each 2D array of the subdomain and the communication between two adjacent subdomains. The data in the darker region is sent to the lighter region of the neighbouring subdomian using PVM message passing calls.

The two most important issues in this implementation are (1) to balance workload (2) to minimize the communication time. In a homogeneous multiprocessor environment, as in our case, the load balancing is assured if all the subdomains are of the same size. Communication is minimized by minimizing the perimeters of the subdomain boundaries.

Conceptually, PVM consists of distributed support software that executes on participating UNIX hosts on a network, allowing them to interconnect and cooperate in a parallel distributed computing environment (Geist et al. 1994). PVM offers an inexpensive platform for developing and running application. Heterogeneous machines can be used in a networked environment. The PVM model is a set of message passing routines which allows data to be exchanged between tasks by sending and receiving messages.

In the PVM implementation of the modelling codes there is a master task and there are number of worker tasks. The main job of master task is to divide the model domain into subdomains and distribute them to worker tasks. The worker tasks perform time marching and communicate after each time step. As demanded by the user the snapshot and synthetic seismogram data are collected by the master and written out on the disk.

NUMERICAL EXAMPLES

The accuracy of the wave modelling algorithms is tested by calculating the wavefield for a corner model shown in Figure 2.

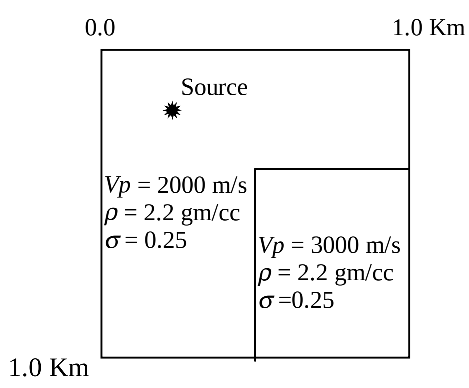


Figure 2: Velocity model for testing the wave propagation algorithms. Vp, ρ and σ denote the P-wave velocity, density and Poisson ratio in the media respectively.

The physical parameters (P-wave velocity, density and Poisson ratio) inside the corner region and outside the corner region are also shown. The source pulse is a band limited Ricker wavelet with a dominant frequency of 30 Hz. The model was discretized using a grid spacing of 2m.

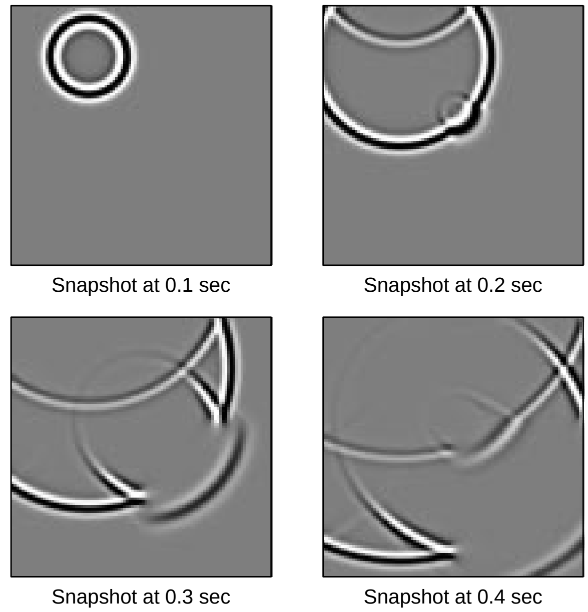


Figure 3: Snapshots of the acoustic wave propagation through a corner model. Free surface boundary condition is used at the top boundary and the absorbing boundary condition is used at all other boundaries.

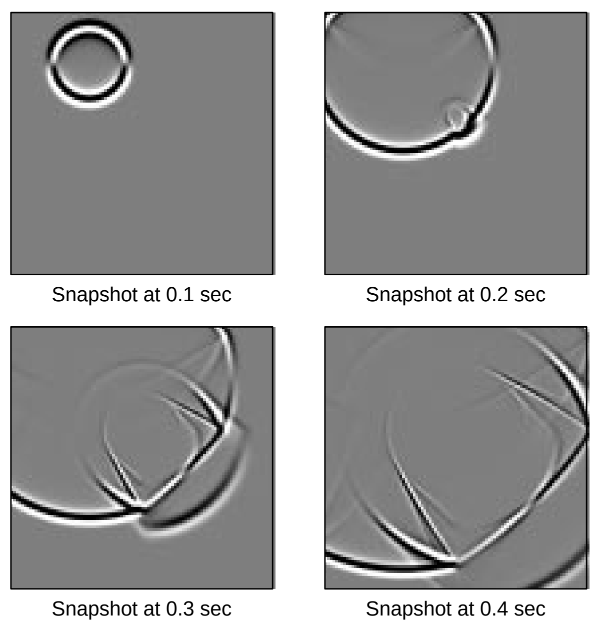


Figure 4: Snapshots of the elastic wave propagation through a corner model. The snapshots show the vertical component of the particle velocity. Absorbing boundary condition is used at all the model boundaries.

Figure 3 shows the snapshots of the acoustic wave propagation in the corner model. On these pictures one can see the direct wave, the wave reflected from the free surface, the wave diffracted from the corner, and the reflected, refracted and transmitted waves. Figure 4 shows the snapshots of the elastic wave propagation. On these snapshots the direct, reflected, refracted, converted and transmitted waves can be easily observed.

DISCUSSION AND CONCLUSIONS

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